Conversion of optical path length to frequency by an interferometer using photorefractive oscillation

Sze-Keung Kwong and Amnon Yariv
California Institute of Technology, Pasadena, California 91125

Mark Cronin-Golomb and Israel Ury
Ortel Corporation, 2015 West Chestnut Street, Alhambra, California 91803

(Received 18 April 1985; accepted for publication 17 June 1985)

Frequency detuning effects in photorefractive oscillators are used in a new type of (passive) interferometry which converts optical path length changes to frequency shifts. Such an interferometer is potentially more accurate than conventional interferometers which convert optical path length changes to phase or intensity changes.

In traditional interferometry, changes in optical path length (OPL) cause changes in fringe position at the output of an interferometer. This change in fringe position is inferred by intensity measuring detectors. The precision of these devices might thus be limited by the precision with which intensity measurements can be made.

Frequency can often be measured with much higher precision than intensity. Therefore, an interferometer whose output can be measured by a frequency counter will benefit from this improved precision. In this letter we report a new class of passive interferometers which can convert OPL to frequency. These devices were suggested by a new theory[1] which explains recently observed frequency detuning effects in certain photorefractive oscillators including the ring resonators[2,3] and the double phase conjugate resonator.[4,5]

In the following, we develop the theory for the frequency shift of the oscillation beam, relative to that of the pump beam, in a photorefractively pumped ring resonator.[6] We then describe the interferometric experiment which demonstrates the theoretical results.

Let us consider the coupled waves theory in a ring oscillator. We use the configuration shown in Fig. 1. We assume that the pumping wave $\bar{E}_1$ and the oscillating wave $\bar{E}_2$ are plane waves and can be written as

$$\bar{E}_j = A_j(\bar{r})\exp[i(\bar{k}_j \cdot \bar{r} - \omega_j t)] + \text{c.c.} \quad j = 1, 2. \tag{1}$$

The interference of these two beams in a photorefractive crystal creates a phase grating inside the crystal. In general there is a nonzero phase shift between the light interference pattern and the phase grating, which will induce energy coupling from one beam to another.[7] Using the standard slowly varying field approximation and the scalar wave equation, we can obtain the following coupled wave equations:

$$\frac{dI_1}{dz} = -\Gamma\frac{I_1 I_2}{I_0} - \alpha I_1, \tag{2a}$$

$$\frac{dI_2}{dz} = \Gamma\frac{I_1 I_2}{I_0} - \alpha I_2, \tag{2b}$$

$$\frac{d\psi_1}{dz} = -\Gamma'\frac{I_2}{I_0}, \tag{2c}$$

$$\frac{d\psi_2}{dz} = -\Gamma'\frac{I_1}{I_0}, \tag{2d}$$

where

$$I_j = |A_j|^2 \text{ and } A_j = |A_j|\exp[i\psi_j] \quad j = 1, 2, \tag{3}$$

$$I_0 = I_1 + I_2,$$
$$\Gamma = 2\text{Re} - (i\omega_0\gamma/2c\cos\theta),\ \Gamma' = \text{Im} - (i\omega_0\gamma/2c\cos\theta), \tag{4}$$

and the coupling constant γ is equal to[8]

$$\gamma = i\gamma_0/[1 + i(\omega - \omega_0)\tau], \tag{5}$$

where we write the pump beam frequency ω_1 as ω_0 and the oscillating beam frequency ω_2 as ω. The constant γ_0 depends on the crystal parameters and the crystal orientation with respect to the two beams, and τ is the photorefractive time constant. α is the intensity absorption coefficient and θ is the angle between the beams and the z axis. Since, as we will show in what follows $(\omega - \omega_0)/\omega_0 \lesssim 10^{-14}$, ω can be accurately replaced by ω_0 except in Eq. (5) so that the magnitudes of coupling strength Γ and Γ' are the same for E_1 and E_2. This approximation will be justified later. We then apply boundary conditions appropriate to a ring oscillator.

$$I_2(0) = RI_2(l), \tag{6}$$

where R is the combined reflectivity for one round trip. From Eqs. (2) and (6), we can solve for $I_2(0)$ and $[\psi_2(l) - \psi_2(0)]$.

$$I_2(0) = I_1(0)\left(\frac{1 - e^{-\Gamma l}}{1 - Re^{-\alpha l}} - 1\right), \tag{7}$$

$$\psi_2(l) - \psi_2(0) = -(\Gamma'/\Gamma)(\alpha l - \ln R). \tag{8}$$

Equation (7) gives the intensity of oscillation, and Eq. (8) gives the phase shift of A_2 due to the nonlinear interaction. From Eqs. (4) and (5), we rewrite Eq. (8) as

$$\psi_2(l) - \psi_2(0) = \tfrac{1}{2}(\omega - \omega_0)\tau(\alpha l - \ln R). \tag{9}$$

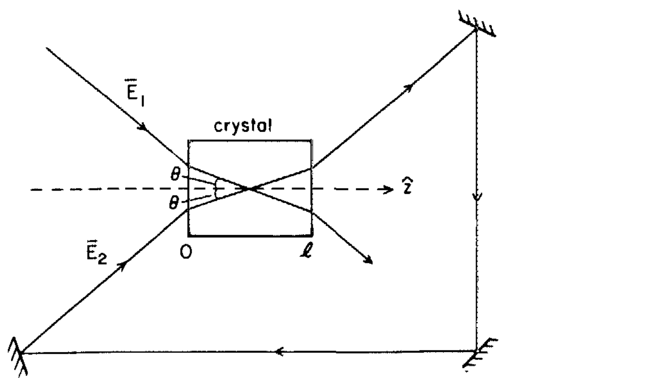

FIG. 1. Schematic diagram of photorefractively pumped unidirectional ring oscillator.

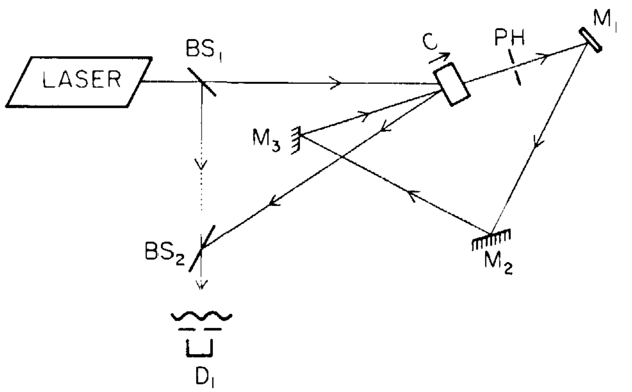

FIG. 2. Experimental configuration for measuring frequency detuning δ vs displacement of mirror M_1. Using a Cartesian coordinate system with the laser beam traveling along the abscissa and the coordinate in inches, the elements are the Ar laser (0, 0); beamsplitters BS_1 (1, 0), BS_2 (1, -3.5); mirrors M_1 (9, 1), M_2 (7, -3), and M_3 (3, -1); crystal C (6, 0); 0.6-mm-diam pinhole PH (7.5, 0.5); detector D_1 (1, -4.5).

The round trip phase condition for mode a is

$$\omega_a L/c = \omega L/c + \psi_2(l) - \psi_2(0), \tag{10}$$

where ω_a is the ath mode frequency of resonator with no photorefractive interaction and L is the length of the resonator. We substitute Eq. (9) into (10) and we get

$$(\omega - \omega_0) = \frac{2L}{c(\alpha l - \ln R)\tau}(\omega_a - \omega)$$

$$= \frac{2t_a}{\tau}(\omega_a - \omega), \tag{11}$$

where t_a is the decay time constant of the photon density in the ath mode. In the limit $t_a \ll \tau$, we can approximate Eq. (18) by

$$(\omega - \omega_0) \cong (2t_a/\tau)(\omega_a - \omega_0). \tag{12}$$

Since for most photorefractive crystals, τ is order of one second and t_a $(\omega_a - \omega_0)$ is roughly equal to 1, so that $\omega - \omega_0$ is order of a few hertz. Therefore, the approximation in Eq. (2), $\omega \cong \omega_0$, is well justified. Equation (12) determines the oscillation frequency, and it is identical to an expression derived using a general oscillator theory [Eq. (17) of Ref. 1]. If we choose the zero detuning, $\omega_a - \omega_0 = 0$, as the origin, we can rewrite the frequency detuning $\delta \equiv (\omega - \omega_0)$ in Eq. (12) as

$$\delta \cong (2t_a/\tau)(\omega_0/L)\Delta L \quad (-\lambda/4 \leqslant \Delta L \leqslant \lambda/4), \tag{13}$$

where ΔL is the mirror displacement from the origin. Equation (13) predicts a linear relationship between the frequency detuning δ to the mirror displacement ΔL.

The experimental arrangement of the OPL to frequency converting interferometer is shown in Fig. 2. The output from a single longitudinal mode argon ion laser ($\lambda = 514.5$ nm, $P = 0.2$ W, beam diameter $= 2$ mm) was directed to the $BaTiO_3$ crystal. The crystal was poled into single domain before the experiment and its c axis was in the plane of the ring oscillator. Mirrors M_1, M_2, and M_3 were aligned to form a ring resonator (38 cm). Mirror M_1 was set on a piezoelectric mount. A 0.6-mm-diam pinhole was inserted inside the oscillating cavity in order to force a stable single mode oscillation (without the pinhole, the oscillation pattern varied erratically in time).[9] A fraction of the oscillating beam reflected from the front surface of the crystal was combined with the pumping laser beam to form interference fringes. Detector D_1 monitored the speed of moving fringes, from which the frequency offset δ was inferred.

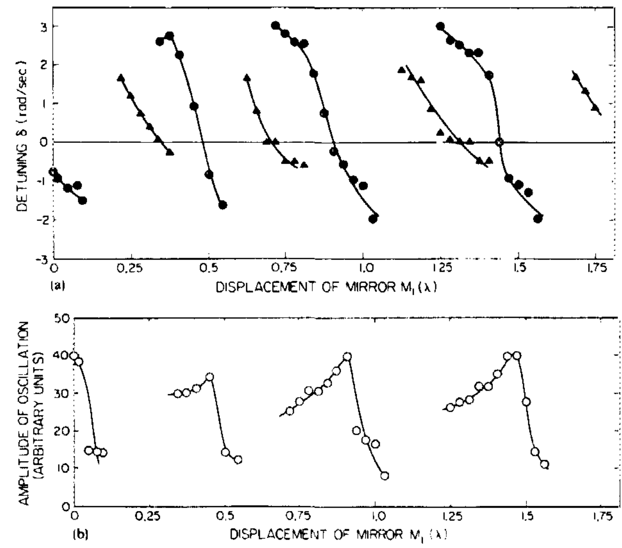

FIG. 3. (a) Experimental data of frequency detuning δ vs displacement of mirror M_1, (●) TEM_{00} mode, (▲) TEM_{01} mode. (b) Oscillating beam power of TEM_{00} mode vs displacement of mirror M_1.

Figure 3(a) shows the frequency detuning δ against the displacement of mirror M_1. We notice that the oscillator can support two transverse modes, TEM_{00} and TEM_{01}. The frequency detuning varied almost linearly with the mirror displacement within each period $\lambda/2$, of mirror displacement. The slight departure from linearity is at least partly due to the fact that τ, appearing in Eq. (13), is inversely proportional to the sum of the pumping and oscillation beam intensities. Since the oscillation intensity is a function of δ and τ [Eqs. (4), (5), and (7), and Fig. 3(b)], the slope of the detuning in Fig. 3(a) is also a function of δ. We also note that the gain linewidth is, according to Eq. (5), $\sim \tau^{-1}$ Hz which in $BaTiO_3$ is a few hertz so that the maximum detuning δ observed is a few hertz. From Fig. 3(a), the slopes of frequency detuning versus the mirror displacement curves give, according to Eq. (13), an estimate of the ratio t_a/τ which is 1.89×10^{-8} and 0.88×10^{-8} for the TEM_{00} and the TEM_{01} modes respectively. This ratio, t_a/τ, is larger for the TEM_{00} mode than the TEM_{01} mode since the latter, suffering higher diffraction losses, has a smaller t_a. At each region of discontinuity in δ, for example, at 0.18λ, 0.6λ, 1.1λ, and 1.65λ, the oscillation was unstable, and rapid mode hopping between the TEM_{00} and TEM_{01} modes was observed. At other points, transitions between TEM_{00} and TEM_{01} could be induced by disturbing the system, for example, by vibrating one of the ring cavity mirrors. The longitudinal mode spacing was 7.9×10^8 Hz and the transverse mode spacing between TEM_{00} and TEM_{01} mode was 2.13×10^8 Hz. The power of the oscillating beam for the TEM_{00} mode was also plotted in Fig. 3(b). The oscillation power was near maximum at zero frequency detuning, $\delta = 0$, which is due to the fact that at this point the coupling constant γ as given by Eq. (5) is maximum.

In summary, we have reported an optical path length to frequency converting interferometer which is based on the frequency pulling effects in photorefractive oscillators. A theory was developed to determine the frequency detuning and oscillation intensity in a ring oscillator. The theory was verified experimentally. Other types of oscillators, such as the linear oscillator and the double phase conjugate resonator,[4,5] can also be used as OPL to frequency converting interferometer.

This work was supported by the U. S. Air Force Office of Scientific Research and the U. S. Army Research Office, Durham, North Carolina.

[1]A. Yariv and S. K. Kwong, Opt. Lett. (to be published).
[2]H. Rajbenbach, J. P. Huignard, Opt. Lett. **10**, 137 (1985).
[3]J. Feinberg and G. D. Bacher, Opt. Lett. **9**, 420 (1984).
[4]M. Cronin-Golomb, B. Fischer, S. K. Kwong, J. O. White, and A. Yariv, Opt. Lett. **10**, 353 (1985).
[5]S. K. Kwong, M. Cronin-Golomb, B. Fischer, and A. Yariv, J. Opt. Soc. Am. B (to be published).
[6]Recently, a similar theoretical analysis on photorefractive coupling of a pair of oppositely directed traveling waves in ring resonators was also reported by P. Yeh, Appl. Opt. **23**, 2974 (1984).
[7]V. L. Vinetskii, N. V. Kukhtarev, S. G. Odulov, and M. S. Soskin, Sov. Phys. Usp. **22**, 742 (1979).
[8]J. P. Huignard and A. Marrackchi, Opt. Commun. **38**, 249 (1981).
[9]G. C. Valley and G. J. Dunning, Opt. Lett. **9**, 513 (1984).